

Kenmore-Town of Tonawanda UFSD

We educate, prepare, and inspire all students to achieve their highest potential



Grade 3 Module 3 Parent Handbook

Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

In this module we will go deep into our learning about these two related operations. Students will practice their math facts to become fluent, and will learn several strategies for multiplying and dividing numbers.



$$2 \times 3 \text{ ones} = 6 \text{ ones}$$

$$2 \times 3 = 6$$



$$2 \times 3 \text{ tens} = 6 \text{ tens}$$

$$2 \times 30 = 60$$

Students will learn to relate simple one-digit facts to similar facts in the place value family.

Key Words to Know

Array: a set of numbers or objects that follow a specific pattern

Commutative Property: e.g. $3 \times 2 = 2 \times 3$

Distributive Property: e.g. $12 \times 3 = (10 + 2) \times 3 = (10 \times 3) + (2 \times 3)$

Factors: numbers that are multiplied to obtain a product
Multiple: e.g. multiples of 9 are 18, 27, 36, 45, etc.

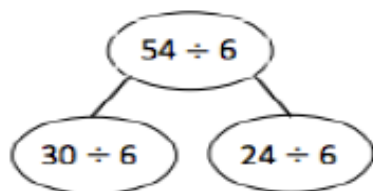
Number bond: model used to show part-part-whole relationships

Product: the quantity resulting from multiplying factors

Quotient: the answer when one number is divided by another

Tape diagram: a method for modeling problems

This is a strategy for division:



Students use facts they already know to help solve an unknown fact.

$$54 \div 6 = (30 \div 6) + (24 \div 6)$$

$$= 5 + 4$$

$$= 9$$

What Came Before this Module: We learned more about both measurement and the place value system. We also worked with telling time to the nearest minute and elapsed time.

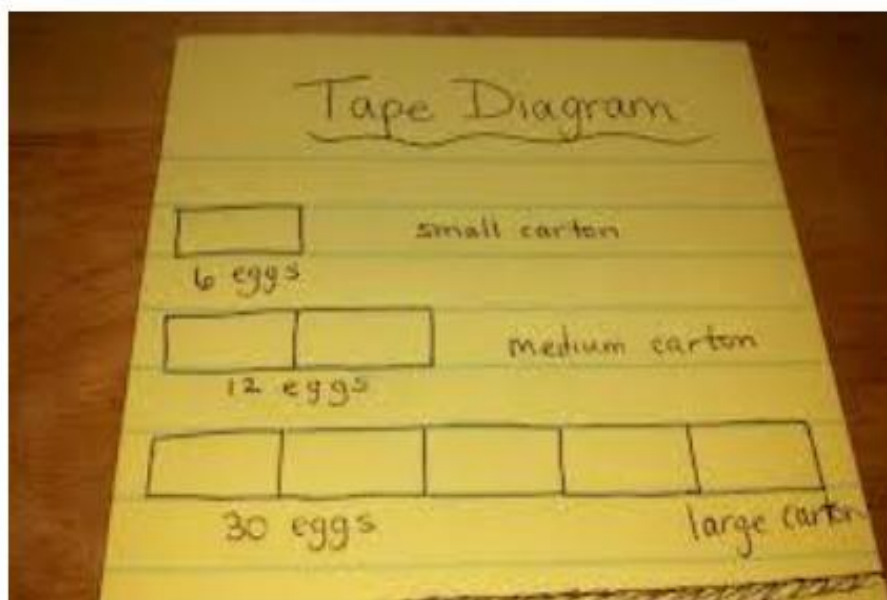
What Comes After this Module: We will extend our multiplication skills by studying area and two-dimensional spaces. We will design a floor plan and calculate the area using our multiplication skills.

+ How you can help at home:

- ⇒ Continue to review multiplication and division math facts with your student
- ⇒ Help your student notice related math facts, e.g. $4 \times 2 = 8$, $4 \times 20 = 80$, $40 \times 2 = 80$

Key Common Core Standards:

- Represent and solve problems involving multiplication and division
- Understand properties of multiplication and the relationship between multiplication and division
- Multiply and divide within 100
- Solve problems involving the four operations
- Use place value understanding and properties of operations to perform multi-digit arithmetic



Spotlight on Math Models:

Tape Diagrams

You will often see this mathematical representation in *A Story of Units*.

A Story of Units has several key mathematical “models” that will be used throughout a student’s elementary years.

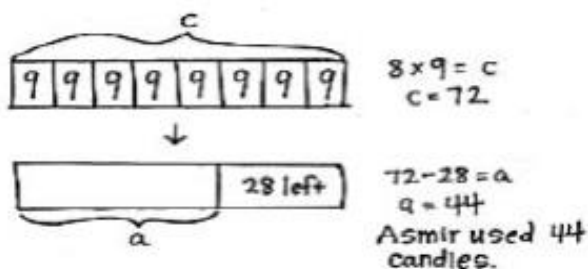
The tape diagram is a powerful model that students can use to solve various kinds of problems. In earlier grades, tape diagrams are models of addition and subtraction, but now in third grade we will use them to model multiplication and division as well. Tape diagrams are also called “bar models” and consist of a simple bar drawing that students make and adjust to fit a word problem. They then use the drawing to discuss and solve the problem.

As students move through the grades, tape diagrams provide an essential bridge to algebra. Below is a sample word problem from Module 3 solved using a tape diagram to show the parts of the problem.

Module 3 Sample Problem

Asmir buys 8 boxes of 9 candles for his dad’s birthday. After putting some candles on the cake, there are 28 candles left. How many candles does Asmir use?

(Example taken from Lesson 11)



Multiplication and Division with Units of 0, 1, 6–9, and Multiples of 10

OVERVIEW

This 25-day module builds directly on students' work with multiplication and division in Module 1. At this point, Module 1 instruction coupled with fluency practice in Module 2 has students well on their way to meeting the Grade 3 fluency expectation for multiplying and dividing within 100 (**3.OA.7**). Module 3 extends the study of factors from 2, 3, 4, 5, and 10 to include all units from 0 to 10, as well as multiples of 10 within 100. Similar to the organization of Module 1, the introduction of new factors in Module 3 spreads across topics. This allows students to build fluency with facts involving a particular unit before moving on. The factors are sequenced to facilitate systematic instruction with increasingly sophisticated strategies and patterns.

Topic A begins by revisiting the commutative property. Students study familiar facts from Module 1 to identify known facts using units of 6, 7, 8, and 9 (**3.OA.5, 3.OA.7**). They realize that they already know more than half of their facts by recognizing, for example, that if they know 2×8 , they also know 8×2 through commutativity. This begins a study of arithmetic patterns that becomes an increasingly prominent theme in the module (**3.OA.9**). The subsequent lesson carries this study a step further; students apply the commutative property to relate 5×8 and 8×5 , and then add one more group of 8 to solve 6×8 and, by extension, 8×6 . The final lesson in this topic builds fluency with familiar multiplication and division facts, preparing students for the work ahead by introducing the use of a letter to represent the unknown in various positions (**3.OA.3, 3.OA.4**).

Topic B introduces units of 6 and 7, factors that are well suited to Level 2 skip-counting strategies and to the Level 3 distributive property strategy, already familiar from Module 1. Students learn to compose up to, then over the next ten. For example, to solve a fact using units of 7 they might count 7, 14, and then mentally add $14 + 6 + 1$ to make 21. This strategy previews the associative property using addition and illuminates arithmetic patterns as students apply count-bys to solve problems (**3.OA.9**). In the next lesson, students apply the distributive property (familiar from Module 1) as a strategy to multiply and divide. They decompose larger unknown facts into smaller known facts to solve. For example, $48 \div 6$ becomes $(30 \div 6) + (18 \div 6)$, or $5 + 3$ (**3.OA.5, 3.OA.7**). Topic B's final lesson emphasizes word problems, providing opportunities to analyze and model. Students apply the skill of using a letter to represent the unknown in various positions within multiplication and division problems (**3.OA.3, 3.OA.4, 3.OA.7**).

Topic C anticipates the formal introduction of the associative property with a lesson focused on making use of structure to problem solve. Students learn the conventional order for performing operations when parentheses are and are not present in an equation (**3.OA.8**). With this student knowledge in place, the associative property emerges in the next lessons as a strategy to multiply using units up to 8 (**3.OA.5**). Units of 6 and 8 are particularly useful for presenting this Level 3 strategy. Rewriting 6 as 2×3 or 8 as 2×4 makes shifts in grouping readily apparent (see example on next page) and also utilizes the familiar factors 2, 3, and 4 as students learn the new material. The following strategy may be used to solve a problem like 8×5 :

$$8 \times 5 = (4 \times 2) \times 5$$

$$8 \times 5 = 4 \times (2 \times 5)$$

$$8 \times 5 = 4 \times 10$$

In the final lesson of Topic C, students relate division to multiplication using units up to 8. They understand division as both a quantity divided into equal groups and an unknown factor problem for which—given the large size of units—skip-counting to solve can be more efficient than dividing (**3.OA.3, 3.OA.4, 3.OA.7**).

Topic D introduces units of 9 over three days, with students exploring a variety of arithmetic patterns that become engaging strategies for quickly learning facts with automaticity (**3.OA.3, 3.OA.7, 3.OA.9**). Nines are placed late in the module so that students have enough experience with multiplication and division to recognize, analyze, and apply the rich patterns found in the manipulation of units of 9. As with other topics, the sequence ends with interpreting the unknown factor to solve multiplication and division problems (**3.OA.3, 3.OA.4, 3.OA.5, 3.OA.7**).

In Topic E, students begin by working with facts using units of 0 and 1. From a procedural standpoint, these are simple facts that require little time for students to master; however, understanding the concept of nothing (zero) is more complex, particularly as it relates to division. This unique combination of simple and complex explains the late introduction of 0 and 1 in the sequence of factors. Students study the results of multiplying and dividing with units of 0 and 1 to identify relationships and patterns (**3.OA.7, 3.OA.9**). The topic closes with a lesson devoted to two-step problems involving all four operations (**3.OA.8**). In this lesson, students work with equations involving unknown quantities and apply the rounding skills learned in Module 2 to make estimations that help them assess the reasonableness of their solutions (**3.OA.8**).

In Topic F, students multiply by multiples of 10 (**3.NBT.3**). To solve a fact like 2×30 , they first model the basic fact 2×3 on the place value chart. Place value understanding helps them to notice that the product shifts one place value to the left when multiplied by 10: 2×3 tens can be found by simply locating the same basic fact in the tens column.

| hundreds | tens | ones |
|----------|------|------------------|
| | | 000 |
| | | 000 |
| | | $2 \times 3 = 6$ |

| hundreds | tens | ones |
|----------|--|------|
| | 000 | |
| | 000 | |
| | $2 \times 3 \text{ tens} = 6 \text{ tens}$ | |
| | $6 \text{ tens} = 60$ | |

In the subsequent lesson, place value understanding becomes more abstract as students model place value strategies using the associative property (**3.NBT.3, 3.OA.5**). $2 \times 30 = 2 \times (3 \times 10) = (2 \times 3) \times 10$. The final lesson focuses on solving two-step word problems involving multiples of 10 and equations with unknown quantities (**3.OA.8**). As in the final lesson of Topic E, students estimate to assess the reasonableness of their solutions (**3.OA.8**).

Terminology

New or Recently Introduced Terms

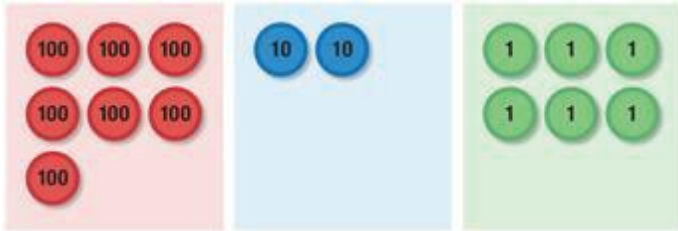
- Multiple (specifically with reference to naming multiples of 9 and 10, e.g., 20, 30, 40, etc.)
- Product (the quantity resulting from multiplying two or more numbers together)

Familiar Terms and Symbols

- Array (a set of numbers or objects that follow a specific pattern)
- Commutative property (e.g., $2 \times 3 = 3 \times 2$)
- Distribute (with reference to the distributive property; e.g., in $12 \times 3 = (10 \times 3) + (2 \times 3)$, the 3 is the multiplier for each part of the decomposition)
- Divide, division (partitioning a total into equal groups to show how many equal groups add up to a specific number, e.g., $15 \div 5 = 3$)
- Equal groups (with reference to multiplication and division; one factor is the number of objects in a group and the other is a multiplier that indicates the number of groups)
- Equation (a statement that two expressions are equal, e.g., $3 \times 4 = 12$)
- Even number (a whole number whose last digit is 0, 2, 4, 6, or 8)
- Expression (a number, or any combination of sums, differences, products, or divisions of numbers that evaluates to a number, e.g., 8×3 , $15 \div 3$)
- Factors (numbers that are multiplied to obtain a product)
- Multiply, multiplication (an operation showing how many times a number is added to itself, e.g., $5 \times 3 = 15$)
- Number bond (model used to show part–part–whole relationships)
- Number sentence (an equation or inequality for which both expressions are numerical and can be evaluated to a single number, e.g., $21 > 7 \times 2$, $5 \div 5 = 1$)
- Odd number (a number that is not even)
- Ones, twos, threes, etc. (units of one, two, or three)
- Parentheses (the symbols () used around a fact or numbers within an equation, expression, or number sentence)
- Quotient (the answer when one number is divided by another)
- Row, column (in reference to rectangular arrays)
- Tape diagram (a method for modeling problems)
- Unit (one segment of a partitioned tape diagram)
- Unknown (the “missing” factor or quantity in multiplication or division)
- Value (how much)

Suggested Tools and Representations

- Array
- Number bond (model used to show part–part–whole relationships)
- Place value disks (pictured below)



- Tape diagram (a method for modeling problems)

Grade 3 Module 3 Topic A

The Properties of Multiplication and Division

Focus Standards:

- 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$*
- 3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
- 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
- 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Instructional Days Recommended: 3

In Lesson 1, students study the commutativity of familiar Module 1 facts that use units of 2, 3, 4, 5, and 10 to discover facts that they already know using units of 6, 7, 8, and 9. For example, students recognize that if they know $3 \times 6 = 18$, then they know $6 \times 3 = 18$. They write out familiar facts and those known through commutativity, organizing them in rows and columns to form the beginning of a table through which arithmetic patterns become visible. Students finish this lesson encouraged about the work to come after seeing that they already know more than half of their facts.

In Lesson 2, students apply commutativity in conjunction with the $n + 1$ strategy to solve unknown facts. For example, students relate 5×8 and 8×5 and then add one more group of 8 to solve 6×8 and by extension, 8×6 . Adding one more group to a known fact in order to find an unknown fact continues to bridge Module 1 and Module 3 learning as students are reminded of their prior work with the distributive property.

Lesson 3 introduces using a letter to represent the unknown in various positions within multiplication and division problems. In Module 1, students represented the unknown on tape diagrams, and occasionally in equations, using a question mark. This lesson uses familiar facts to introduce the new abstraction of using a letter as a placeholder.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 1

Objective: Study commutativity to find known facts of 6, 7, 8, and 9.

Homework Key

- a. 9; 4; 15; 6; 21
b. 5; 6; 28; 32; 36
c. 30; 7; 8; 9; 50
- 24, 4, 6; 24, 6, 4
24, 3, 8; 24, 8, 3
- Expressions accurately matched
- a. 6
b. 3; 18
c. 8; 32
d. 7; 7
e. 7, 2; 14
f. 5; 30

Homework Samples

- Complete the charts below.

- a. A tricycle has 3 wheels.

| | | | | | |
|------------------------|---|----|----|----|----|
| Number of Tricycles | 3 | 4 | 5 | 6 | 7 |
| Total Number of Wheels | 9 | 12 | 15 | 18 | 21 |

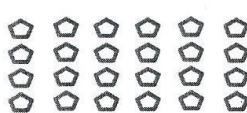
- b. A tiger has 4 legs.


| | | | | | |
|----------------------|----|----|----|----|----|
| Number of Tigers | 5 | 6 | 7 | 8 | 9 |
| Total Number of Legs | 20 | 24 | 28 | 32 | 36 |

- c. A package has 5 erasers.

| | | | | | |
|-------------------------|----|----|----|----|----|
| Number of Packages | 6 | 7 | 8 | 9 | 10 |
| Total Number of Erasers | 30 | 35 | 40 | 45 | 50 |

- Write two multiplication facts for each array.


$$\begin{aligned} 24 &= 6 \times 4 \\ 24 &= 4 \times 6 \end{aligned}$$


$$\begin{aligned} 24 &= 3 \times 8 \\ 24 &= 8 \times 3 \end{aligned}$$

4. Complete the equations.

a. 2 sixes = 6 twos
= 12

Lesson 2

Objective: Apply the distributive and commutative properties to relate multiplication facts $5 \times n + n$ to $6 \times n$ and $n \times 6$ where n is the size of the unit.

Homework Key

- 5 nines, 9, 9, 45
5, 1, 9, 54; 6, 9, 54; 9, 6, 54
- 42
- 6
- 3

Homework Samples

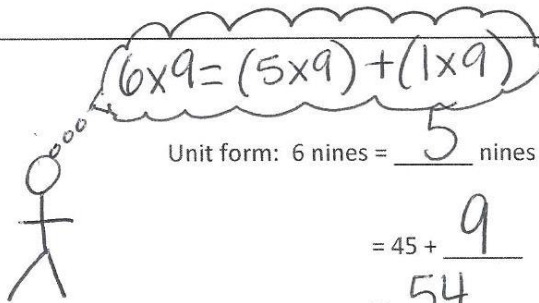
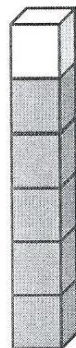
1. Each  has a value of 9.



Unit form: 5 nines

Facts: $5 \times \underline{9} = \underline{9} \times 5$

Total = 45



Unit form: 6 nines = 5 nines + 1 nine

= 45 + 9
= 54

Facts: $\underline{6} \times \underline{9} = \underline{54}$

$\underline{9} \times \underline{6} = \underline{54}$

Lesson 3

Objective: Multiply and divide with familiar facts using a letter to represent the unknown.

Homework Key

- 40, 50, 70, 80, 100
 - $e = 30$; $f = 40$; $p = 50$; $w = 60$; $n = 70$; $g = 80$
- $n = 4$; $a = 4$; $p = 5$; $c = 3$; $d = 6$; $h = 35$; $f = 18$; $y = 8$
- $b = \$28$
 - $c = \$2$; answers will vary.
- 50 m; answers will vary.

Homework Samples

- Complete the pattern.



- Find the value of the unknown.

| | | | |
|-------------------|----------------------|-------------------|----------------------|
| $10 \times 2 = d$ | $d = \underline{20}$ | $10 \times 6 = w$ | $w = \underline{60}$ |
| $3 \times 10 = e$ | $e = \underline{30}$ | $10 \times 7 = n$ | $n = \underline{70}$ |
| $f = 4 \times 10$ | $f = \underline{40}$ | $g = 8 \times 10$ | $g = \underline{80}$ |
| $p = 5 \times 10$ | $p = \underline{50}$ | | |

- Each equation contains a letter representing the unknown. Find the value of the unknown.

| | |
|-------------------|----------------------|
| $8 \div 2 = n$ | $n = \underline{4}$ |
| $3 \times a = 12$ | $a = \underline{4}$ |
| $p \times 8 = 40$ | $p = \underline{5}$ |
| $18 \div 6 = c$ | $c = \underline{3}$ |
| $d \times 4 = 24$ | $d = \underline{6}$ |
| $h \div 7 = 5$ | $h = \underline{35}$ |
| $6 \times 3 = f$ | $f = \underline{18}$ |
| $32 \div y = 4$ | $y = \underline{8}$ |

Grade 3 Module 3 Topic B

Multiplication and Division Using Units of 6 and 7

Focus Standards:

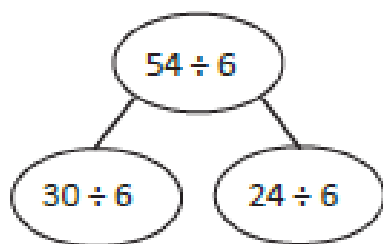
- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$*
- 3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
- 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Instructional Days Recommended: 4

In Lessons 4 and 5, students count by sixes and sevens, composing up to and then over the next ten. For example, students might count 6, 12, 18 and then mentally add $18 + 2 + 4$ to make 24. This skip-counting method utilizes make ten strategies from Grades 1 and 2. Initially, students use number bonds to decompose and identify appropriate number pairs. In the example above, 18 needs 2 to more to make 20. The next six can be decomposed as 2 and 4. Eventually, students are able to use mental math as they manipulate numbers and skip-count to multiply. Although a formal introduction to the associative property comes in Topic C, these lessons preview the concept using addition:

- $6 + 6 = 6 + 4 + 2$
- $18 + 6 = 18 + 4 + 2$
- $36 + 6 = 36 + 4 + 2$
- $48 + 6 = 48 + 4 + 2$

Lesson 6 builds on Lesson 2 with a formal re-introduction of the distributive property using the $5 + n$ pattern to multiply and divide. Students understand that multiples of 6 can be thought of as $(5 + 1) \times n$ to make 5 and 1 more group, or 6 groups of n . Similarly, multiples of 7 can be thought of as $(5 + 2) \times n$ to make 5 and 2 more groups, or 7 groups of n . In division, students decompose the dividend using a multiple of 5 and then add the quotients of the smaller division facts to find the quotient of the larger unknown division fact. For example:



$$\begin{aligned}
 54 \div 6 &= (30 \div 6) + (24 \div 6) \\
 &= 5 + 4 \\
 &= 9
 \end{aligned}$$

Use of the $5 + n$ pattern as a strategy builds on concepts in Lessons 2, 4, and 5. It also facilitates mental math, particularly using units of 6.

In Lesson 7, students use tape diagrams to analyze multiplication and division word problems and to determine the unknown. This is the first time they solve problems using new units, with a letter to represent the unknown.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 4

Objective: Count by units of 6 to multiply and divide using number bonds to decompose.

Homework Key

- 12
 - 18
 - 20, 4, 24
 - 20, 10, 30
 - 36
 - 40, 2, 42
 - Answers will vary; 48
 - Answers will vary; 54
 - Answers will vary; 60
- 12, 18, 24, 30; 5, 30; 30, 5
- 12, 18, 24, 30, 36; 6, 36; 36, 6
- 8; answers will vary.

Homework Samples

- Use number bonds to help you skip-count by six by either making a ten or adding to the ones.

| |
|--|
| a. $6 + 6 = \underline{10} + \underline{2} = \underline{12}$ |
| b. $12 + 6 = \underline{10} + \underline{8} = \underline{18}$ |
| c. $18 + 6 = \underline{20} + \underline{4} = \underline{24}$ |
| d. $24 + 6 = \underline{20} + \underline{10} = \underline{30}$ |
| e. $30 + 6 = \underline{36}$ |
| f. $36 + 6 = \underline{40} + \underline{2} = \underline{42}$ |
| g. $42 + 6 = \underline{40} + \underline{8} = \underline{48}$ |
| h. $48 + 6 = \underline{50} + \underline{4} = \underline{54}$ |
| i. $54 + 6 = \underline{50} + \underline{10} = \underline{60}$ |

Lesson 5

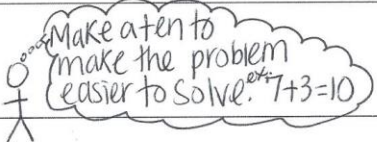
Objective: Count by units of 7 to multiply and divide using number bonds to decompose.

Homework Key

- 14
 - 20, 1, 21
 - 20, 8, 28
 - 30, 5, 35
 - 40, 2, 42
 - 40, 9, 49
 - 50, 6, 56; answers may vary.
 - 60, 3, 63; answers may vary.
- 70, 63, 56, 42, 35, 21, 14
70, 63, 56, 49, 42, 35, 28, 21, 14, 7
70, 10; 63, 9; 56, 8; 49, 7; 42, 6; 35, 5; 28, 4; 21, 3; 14, 2; 7, 1

Homework Samples

- Use number bonds to help you skip-count by seven by making ten or adding to the ones.

| | |
|--|---|
| a. $\begin{array}{r} 7 \\ 3 \end{array} + 7 = \frac{10}{4} + 4 = 14$ |  |
| b. $\begin{array}{r} 14 \\ 6 \end{array} + 7 = \frac{20}{1} + 1 = 21$ | |
| c. $\begin{array}{r} 21 \\ 20 \end{array} + 7 = \frac{20}{1} + 8 = 28$ | |
| d. $\begin{array}{r} 28 \\ 2 \end{array} + 7 = \frac{30}{5} + 5 = 35$ | |
| e. $\begin{array}{r} 35 \\ 5 \end{array} + 7 = \frac{40}{2} + 2 = 42$ | |
| f. $\begin{array}{r} 42 \\ 40 \end{array} + 7 = \frac{40}{2} + 9 = 49$ | |
| g. $\begin{array}{r} 49 \\ 1 \end{array} + 7 = \frac{50}{6} + 6 = 56$ | |
| h. $\begin{array}{r} 56 \\ 4 \end{array} + 7 = \frac{60}{3} + 3 = 63$ | |

Lesson 6

Objective: Use the distributive property as a strategy to multiply and divide using units of 6 and 7.

Homework Key

- Tape diagrams accurately labeled; 42; 35; 1, 7; 7, 42
 - Tape diagrams accurately labeled; 49; 35; 2, 14; 14, 49
 - Tape diagrams accurately labeled; 56; 35; 3, 21; 3; 3; 21; 56
 - Tape diagrams accurately labeled; 63; 35; 4, 28; 4; 4; 28; 63
- 24; 24; 4; 9
- 21; 35, 7; 21, 7; 3; 8
- 7; explanations will vary.
- Yes; explanations will vary.

Homework Samples

- Label the tape diagrams. Then, fill in the blanks below to make the statements true.

a. $6 \times 7 = \underline{42}$

$(5 \times 7) = \underline{35}$
 5×7

$(\underline{1}) \times 7 = \underline{7}$
 1×7

$6 \times 7 = (5 \times 7) + (1 \times 7)$

$(6 \times 7) = (5 + 1) \times 7$
 $= (5 \times 7) + (1 \times 7)$
 $= \underline{35} + \underline{7}$
 $= \underline{42}$

b. $7 \times 7 = \underline{49}$

$(5 \times 7) = \underline{35}$
 5×7

$(\underline{2}) \times 7 = \underline{14}$
 2×7

$7 \times 7 = (5 \times 7) + (2 \times 7)$

$7 \times 7 = (5 + 2) \times 7$
 $= (5 \times 7) + (2 \times 7)$
 $= \underline{35} + \underline{14}$
 $= \underline{49}$

Lesson 7

Objective: Interpret the unknown in multiplication and division to model and solve problems using units of 6 and 7.

Homework Key

- Words matched to corresponding equations
- $m = \$42$; tape diagram drawn and labeled; equations may vary.
 - $p = 36$; tape diagram drawn and labeled; equations may vary.
- $n = 4$; tape diagram drawn and labeled; equations may vary.

Homework Samples

- Match the words on the arrow to the correct equation on the target.

7 times a number equals 42

63 divided by a number equals 9

36 divided by a number equals 6

A number times 7 equals 21

$n \times 7 = 21$

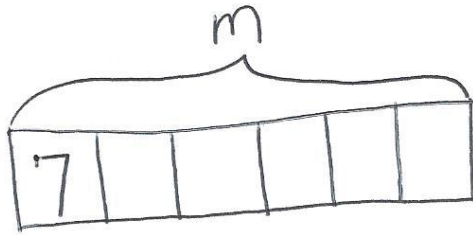
$7 \times n = 42$

$63 \div n = 9$

$36 \div n = 6$

2. Ari sells 6 boxes of pens at the school store.

- a. Each box of pens sells for \$7. Draw a tape diagram and label the total amount of money he makes as m . Write an equation and solve for m .

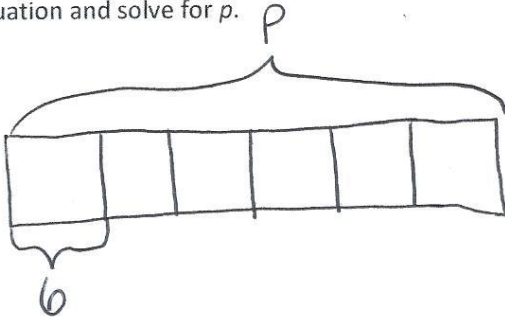


$$6 \times 7 = m$$

$$42 = m$$

Ari makes \$42 selling boxes of pens.

- b. Each box contains 6 pens. Draw a tape diagram and label the total number of pens as p . Write an equation and solve for p .



$$6 \times 6 = p$$

$$p = 36$$

There are 36 pens.

Grade 3 Module 3 Topic C

Multiplication and Division Using Units of up to 8

Focus Standards:

- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$*
- 3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
- 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.

Instructional Days Recommended: 4

Students are informally familiar with parentheses from having seen them in distributive property lessons in Topic B and in Module 1. In Lesson 8, they understand parentheses as tools for grouping and learn the conventional order for performing Grade 3 operations. This practice anticipates applying parentheses in Lesson 9 as students formally study the associative property.

In Lesson 9, students model and demonstrate how to multiplicatively compose or decompose to make problems using units up to 8 easier to solve. For example, 8×5 may be thought of as:

$$\begin{aligned} 8 \times 5 &= (4 \times 2) \times 5 \\ &= 4 \times (2 \times 5) \\ &= 4 \times 10 \end{aligned}$$

Lessons 10 and 11 in this topic parallel Lessons 6 and 7 in Topic B. In Lesson 10, students use the $5 + n$ pattern as a strategy for solving multiplication and division problems using units of 8 with the distributive property. They learn that multiples of 8 can be thought of as $(5 + 3) \times n$. In division problems, students practice decomposing the dividend using multiples of 5. They recognize the efficacy of using this strategy when the quotient of a division equation is greater than 5 and also realize that the dividend must be decomposed into numbers that are divisible by the divisor. For example, to solve $64 \div 8$, 64 can be decomposed as 40 and 24 because both are divisible by 8.

In Lesson 11, students analyze, model, and solve multiplication and division word problems using units of 8. They understand division as both a quantity divided into equal groups, as well as an unknown factor problem. They draw models and write equations to interpret and solve problems, using a letter to represent the unknown in various positions.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 8

Objective: Understand the function of parentheses and apply to solving problems.

Homework Key

- 0
 - 6
 - 8
 - 12
 - 42
 - 22
 - 12
 - 2
- $14 - (8 + 2) = 4$
 - $(14 - 8) + 2 = 8$
 - $2 + (4 \times 7) = 30$
 - $(2 + 4) \times 7 = 42$
 - $12 = (18 \div 3) \times 2$
 - $3 = 18 \div (3 \times 2)$
 - $5 = 50 \div (5 \times 2)$
 - $20 = (50 \div 5) \times 2$
- Answer provided
 - True
 - False
 - True
 - False
- Explanations may vary.
- $(4 \times 7) - 3 = 25$
- Agree; answers will vary.

Homework Samples

1. Solve.

a. $9 - (6 + 3) = \underline{0}$

b. $(9 - 6) + 3 = \underline{6}$

c. $\underline{8} = 14 - (4 + 2)$

d. $\underline{12} = (14 - 4) + 2$

e. $\underline{42} = (4 + 3) \times 6$

f. $\underline{22} = 4 + (3 \times 6)$

g. $(18 \div 3) + 6 = \underline{12}$

h. $18 \div (3 + 6) = \underline{2}$

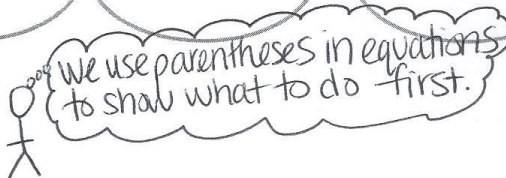
2. Use parentheses to make the equations true.

a. $14 - (8 + 2) = 4$

b. $(14 - 8) + 2 = 8$

c. $2 + (4 \times 7) = 30$

d. $(2 + 4) \times 7 = 42$


We use parentheses in equations to show what to do first.

e. $12 = (18 \div 3) \times 2$

f. $3 = 18 \div (3 \times 2)$

g. $5 = 50 \div (5 \times 2)$

h. $20 = (50 \div 5) \times 2$

Lesson 9

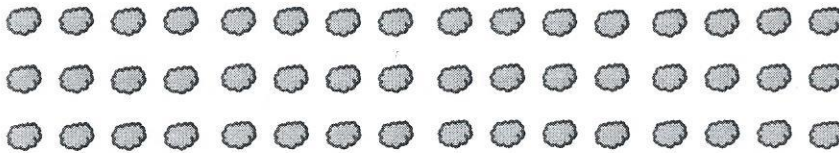
Objective: Model the associative property as a strategy to multiply.

Homework Key

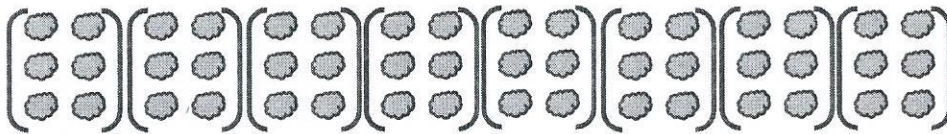
- 48
 - 2; 6, 8; 48
 - 72
 - 2; 8, 9; 72
- 6, 42
 - 9, 36
- Answer provided.
 - $60; 6 \times (5 \times 2)$
 - $70; 7 \times (5 \times 2)$
 - $80; 8 \times (5 \times 2)$

Homework Samples

- Use the array to complete the equation.



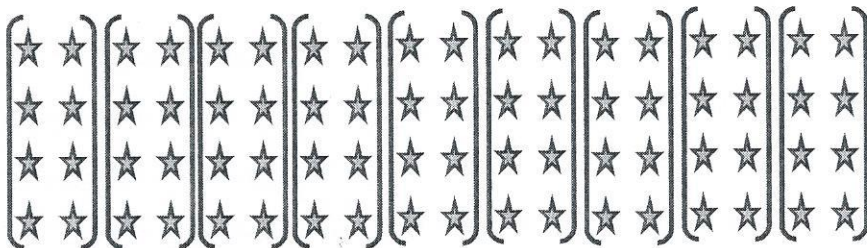
a. $3 \times 16 = \underline{48}$



b. $(3 \times \underline{2}) \times 8$
 $= \underline{6} \times \underline{8}$
 $= \underline{48}$



c. $4 \times 18 = \underline{72}$



d. $(4 \times \underline{2}) \times 9$
 $= \underline{8} \times \underline{9}$
 $= \underline{72}$

Lesson 10

Objective: Use the distributive property as a strategy to multiply and divide.

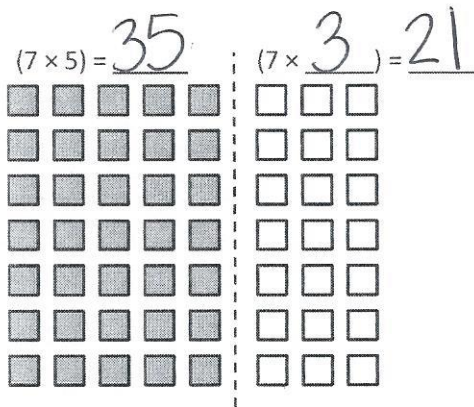
Homework Key

1. Array accurately labeled; 56; 35; 3, 21; 3; 3; 21; 56
2. 32; 4; 9
3. 16, 24, 32, 40, 48, 56, 64, 72, 80; 72, 40, 64, 48, 56
4. 2; 5; 4; 6; 7; 9

Homework Samples

1. Label the array. Then, fill in the blanks to make the statements true.

$$8 \times 7 = 7 \times 8 = \underline{56}$$



$$\begin{aligned} 8 \times 7 &= 7 \times (5 + \underline{3}) \\ &= (7 \times 5) + (7 \times \underline{3}) \\ &= \underline{35} + \underline{21} \\ &= \underline{56} \end{aligned}$$

Lesson 11

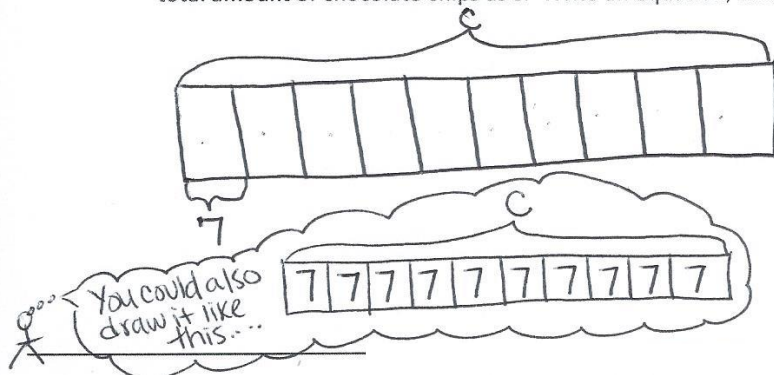
Objective: Interpret the unknown in multiplication and division to model and solve problems.

Homework Key

1. Tape diagram drawn and labeled; $c = 70$
2. Tape diagram drawn and labeled; $v = 6$
3. $m = 7$
4. 54
5. 10
6. \$18

Homework Samples

1. Jenny bakes 10 cookies. She puts 7 chocolate chips on each cookie. Draw a tape diagram, and label the total amount of chocolate chips as c . Write an equation, and solve for c .



$$10 \times 7 = c$$
$$70 = c$$

There are 70
chocolate chips.

Grade 3 Module 3 Topic D

Multiplication and Division Using Units of 9

Focus Standard:

- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48$, $5 = _ \div 3$, $6 \times 6 = ?$*
- 3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
- 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
- 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of

operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Instructional Days Recommended: 4

In Lesson 12, students use the distributive property to establish the $9 = 10 - 1$ pattern for multiplication. Conceptual understanding of the pattern enables students to see this method of multiplication as a tool rather than a trick. This lesson lays the foundation for exploring other patterns that emerge with multiplication using units of 9 in the subsequent lessons.

Lessons 13 and 14 focus on the study of patterns as they relate to the fact $9 = 10 - 1$. Students discover that the tens digit in the product of a nines fact is 1 less than the multiplier and that the ones digit in the product is 10 minus the multiplier. For example, $9 \times 3 = 27$, $2 = 3 - 1$, and $7 = 10 - 3$. They also see that the digits of nines facts' products produce a sum of 9, as in the example above ($2 + 7 = 9$).

Lesson 15 parallels the final lessons of Topics B and C. Students analyze multiplication and division problems using units of 9, drawing models and writing equations using a letter to represent the unknown. These lessons are intended to provide students with continuous experience relating three numbers to find the unknown, as well as to deepen their understanding of the relationship between multiplication and division.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 12

Objective: Apply the distributive property and the fact $9 = 10 - 1$ as a strategy to multiply.

Homework Key

1.
 - a. 54; 24; 24; 54
 - b. 63; 35; 4, 28; 4; 4; 28; 63
 - c. 72; 40; 4, 32; 4; 4, 8; 32; 72
 - d. 81; 45; 4, 36; 4; 4, 9; 36; 81
2.
 - a. Answer provided
 - b. 60; 54; 9×6
 - c. 70; 63; 9×7
 - d. 80; 72; 9×8
 - e. 90, 9; 81; 9×9
 - f. 40, 4; 36; 9×4

Homework Samples

1. Find the value of each row. Then, add the rows to find the total.

a. Each  has a value of 6.

$$9 \times 6 = \underline{54}$$

 $5 \times 6 = 30$

 $4 \times 6 = \underline{24}$

$$\begin{aligned} 9 \times 6 &= (5 + 4) \times 6 \\ &= (5 \times 6) + (4 \times 6) \\ &= 30 + \underline{24} \\ &= \underline{54} \end{aligned}$$

b. Each  has a value of 7.

$$9 \times 7 = \underline{63}$$

 $5 \times 7 = \underline{35}$

 $4 \times 7 = \underline{28}$

$$\begin{aligned} 9 \times 7 &= (5 + \underline{4}) \times 7 \\ &= (5 \times 7) + (\underline{4} \times 7) \\ &= 35 + \underline{28} \\ &= \underline{63} \end{aligned}$$

c. Each  has a value of 8.

$$9 \times 8 = \underline{72}$$

 $5 \times 8 = \underline{40}$

 $4 \times 8 = \underline{32}$

$$\begin{aligned} 9 \times 8 &= (5 + \underline{4}) \times 8 \\ &= (5 \times 8) + (\underline{4} \times \underline{8}) \\ &= 40 + \underline{32} \\ &= \underline{72} \end{aligned}$$

d. Each  has a value of 9.

$$9 \times 9 = \underline{81}$$

 $5 \times 9 = \underline{45}$

 $4 \times 9 = \underline{36}$

$$\begin{aligned} 9 \times 9 &= (5 + \underline{4}) \times 9 \\ &= (5 \times 9) + (\underline{4} \times \underline{9}) \\ &= 45 + \underline{36} \\ &= \underline{81} \end{aligned}$$

Lesson 13 - 14

Objective: Identify and use arithmetic patterns to multiply.

Homework Key (13)

- 81, 63, 54, 45, 27, 18, 9
 - 1
 - +1
- $a = 2; m = 27; e = 5; f = 36; d = 9; w = 54; s = 10;$
 $k = 72$
- 10; 9; 9
 - 19; 18; 18
 - 28; 27; 27
 - 37; 36; 36
 - 46; 45; 45
 - 55; 54; 54
 - 64; 63; 63
 - 73; 72; 72
 - 82; 81; 81
 - 91; 90; 90
- Answers will vary.; 99; 108; 117

Homework Samples

- Skip-count by nines down from 90.

90, 81, 72, 63, 54, 45, 36, 27, 18, 9

- Look at the *tens* place in the count-by. What is the pattern?

It goes down by 1 ten ex. $9 \rightarrow 8 \rightarrow 7, \dots$

- Look at the *ones* place in the count-by. What is the pattern?

It goes up by 1 ex. $0 \rightarrow 1 \rightarrow 2, \dots$

Lesson 14

Homework Key

1. a. Answer provided

Answer provided

72, 7, 2, 9

63, 6, 3, 9

54, 5, 4, 9

45, 4, 5, 9

36, 3, 6, 9

27, 2, 7, 9

18, 1, 8, 9

9, 0, 9, 9

b. Answers will vary.

2. Answers will vary.

3. 54; explanations will vary.

4. Correct; answers will vary.

Homework Samples

1. a. Multiply. Then, add the digits in each product.

| | |
|--------------------|---|
| $10 \times 9 = 90$ | $\underline{9} + \underline{0} = \underline{9}$ |
| $9 \times 9 = 81$ | $\underline{8} + \underline{1} = \underline{9}$ |
| $8 \times 9 = 72$ | $\underline{7} + \underline{2} = \underline{9}$ |
| $7 \times 9 = 63$ | $\underline{6} + \underline{3} = \underline{9}$ |
| $6 \times 9 = 54$ | $\underline{5} + \underline{4} = \underline{9}$ |
| $5 \times 9 = 45$ | $\underline{4} + \underline{5} = \underline{9}$ |
| $4 \times 9 = 36$ | $\underline{3} + \underline{6} = \underline{9}$ |
| $3 \times 9 = 27$ | $\underline{2} + \underline{7} = \underline{9}$ |
| $2 \times 9 = 18$ | $\underline{1} + \underline{8} = \underline{9}$ |
| $1 \times 9 = 9$ | $\underline{0} + \underline{9} = \underline{9}$ |

b. What pattern did you notice in Problem 1(a)? How can this strategy help you check your work with nines facts?

The digit in the tens place plus the digit in the ones place equals 9. I can check my $\times 9$ facts by making sure the sum of the tens and ones place equals 9.

Lesson 15

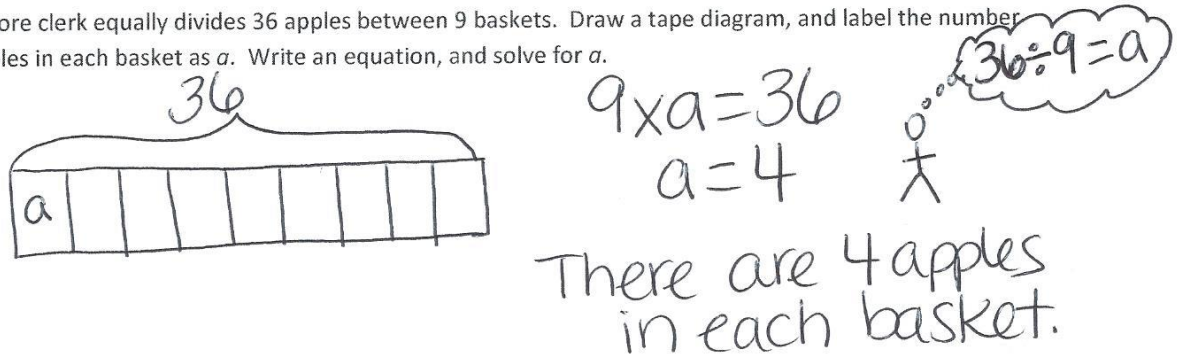
Objective: Interpret the unknown in multiplication and division to model and solve problems.

Homework Key

1. Tape diagram drawn and labeled; $36 \div 9 = a$; $a = 4$
2. 5; solution includes an unknown
3. \$63; solution includes an unknown
4. 9 m
5. 54
6. 3

Homework Samples

1. The store clerk equally divides 36 apples between 9 baskets. Draw a tape diagram, and label the number of apples in each basket as a . Write an equation, and solve for a .



Handwritten student work for a division problem:

A tape diagram shows a total of 36 divided into 9 equal sections, with the first section labeled a .

Equations written: $9 \times a = 36$ and $a = 4$.

A stick figure has a thought bubble containing the equation $36 \div 9 = a$.

Text written: "There are 4 apples in each basket."

Grade 3 Module 3 Topic E

Analysis of Patterns and Problem Solving Including Units of 0 and 1

Focus Standards:

- 3.OA.3 Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.
- 3.OA.7 Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows $40 \div 5 = 8$) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
- 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)
- 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*

Instructional Days Recommended: 3

In Lesson 16, students multiply and divide by 0 and 1. They use patterns to understand that $n \times 0 = 0$ and show why the result of dividing a number by 0 is undefined but that dividing 0 by another number results in 0. Lesson 17 synthesizes students' knowledge of factors from 0 to 10 in an exploration of patterns using the multiplication table. Students recognize the patterns of particular factors and make connections between multiplication and division.

In Lesson 18, students apply the tools, representations, and concepts they have learned in order to solve two-step word problems using all four operations. They call on rounding skills learned in Module 2 to estimate solutions and use their estimations to assess the reasonableness of answers.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 16

Objective: Reason about and explain arithmetic patterns using units of 0 and 1 as they relate to multiplication and division.

Homework Key

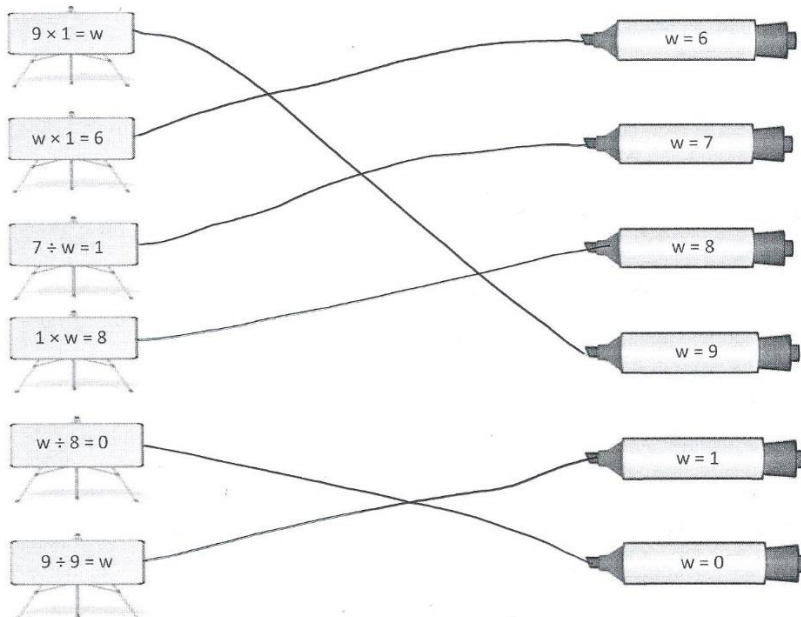
- | | |
|---------|-----------------------------------|
| 1. a. 4 | 2. Equations matched to solutions |
| b. 0 | 3. a. Answer provided |
| c. 5 | b. True |
| d. 0 | c. True |
| e. 1 | d. True |
| f. 0 | e. False |
| g. 0 | f. True |
| h. 0 | g. True |
| i. 1 | h. False |
| j. 1 | 4. a. $n \times 1 = n$ |
| k. 1 | b. Answers will vary. |
| l. 9 | |

Homework Samples

1. Complete.

- | | | | |
|---------------------------------|---------------------------------|---------------------------------|---------------------------------|
| a. $4 \times 1 = \underline{4}$ | b. $4 \times 0 = \underline{0}$ | c. $\underline{5} \times 1 = 5$ | d. $\underline{0} \div 5 = 0$ |
| e. $6 \times \underline{1} = 6$ | f. $\underline{0} \div 6 = 0$ | g. $0 \div 7 = \underline{0}$ | h. $7 \times \underline{0} = 0$ |
| i. $8 \div \underline{1} = 8$ | j. $\underline{1} \times 8 = 8$ | k. $9 \times \underline{1} = 9$ | l. $9 \div \underline{9} = 1$ |

2. Match each equation with its solution.



Lesson 17

Objective: Identify patterns in multiplication and division facts using the multiplication table.

Homework Key

1. a. Products accurately recorded
 - b. Even factors accurately identified
 - c. Explanations may vary.
 - d. Odd; even; even; examples will vary.
 - e. Explanations may vary.
 - f. 64
2. a. Answer provided
 - b. $16 = 4 \times 4$
 - c. $36 = 6 \times 6$
 - d. $64 = 8 \times 8$
 - e. $100 = 10 \times 10$

Homework Samples

1. a. Write the products into the chart as fast as you can.

| | | | | | | | | |
|---|---|----|----|----|----|----|----|----|
| x | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 1 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
| 2 | 2 | 4 | 6 | 8 | 10 | 12 | 14 | 16 |
| 3 | 3 | 6 | 9 | 12 | 15 | 18 | 21 | 24 |
| 4 | 4 | 8 | 12 | 16 | 20 | 24 | 28 | 32 |
| 5 | 5 | 10 | 15 | 20 | 25 | 30 | 35 | 40 |
| 6 | 6 | 12 | 18 | 24 | 30 | 36 | 42 | 48 |
| 7 | 7 | 14 | 21 | 28 | 35 | 42 | 49 | 56 |
| 8 | 8 | 16 | 24 | 32 | 40 | 48 | 56 | 64 |

- b. Color the rows and columns with even factors yellow.

- c. What do you notice about the factors and products that are left unshaded?

The factors and products unshaded are all odd.

- d. Complete the chart below by filling in each blank and writing an example for each rule.



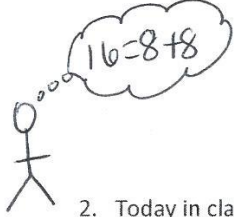
odd x odd = odd

| Rule | Example |
|------------------------------------|--|
| odd times odd equals <u>odd</u> | $3 \times 3 = 9$ $7 \times 5 = 35$ |
| even times even equals <u>even</u> | $6 \times 4 = 24$ $8 \times 2 = 16$ |
| even times odd equals <u>even</u> | $5 \times 2 = 10$ $6 \times 3 = 18$ |

e. Explain how $7 \times 6 = (5 \times 6) + (2 \times 6)$ is shown in the table.

The table shows $5 \times 6 = 30$ and $2 \times 6 = 12$. So,
 $30 + 12 = 42$ which is the product of 7×6 .

f. Use what you know to find the product of 4×16 or 8 fours + 8 fours.



$$\begin{aligned} 4 \times 16 &= (4 \times 8) + (4 \times 8) \\ &= 32 + 32 \\ &= 64 \end{aligned}$$

2. Today in class, we found that $n \times n$ is the sum of the first n odd numbers. Use this pattern to find the

Lesson 18

Objective: Solve two-step word problems involving all four operations and assess the reasonableness of solutions.

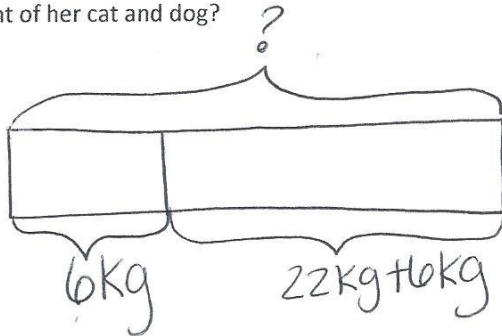
Homework Key

1. 34 kg; solution includes model, equation, and explanation.
2. 57 min; solution includes model, equation, and explanation.
3. 33; solution includes model, equation, and explanation.
4. 7; solution includes model, equation, and explanation.
5. 8 cm; solution includes model, equation, and explanation.
6. \$8; solution includes model, equation, and explanation.

Homework Samples

Use the RDW process for each problem. Explain why your answer is reasonable.

1. Mrs. Portillo's cat weighs 6 kilograms. Her dog weighs 22 kilograms more than her cat. What is the total weight of her cat and dog?



$$6\text{kg} + (22\text{kg} + 6\text{kg}) = W$$
$$6\text{kg} + 28\text{kg} = W$$
$$34\text{kg} = W$$

The total weight of her cat & dog is 34kg.

Grade 3 Module 3 Topic F

Multiplication of Single-Digit Factors and Multiples of 10

Focus Standards:

- 3.OA.5 Apply properties of operations as strategies to multiply and divide. (Students need not use formal terms for these properties.)
Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)
- 3.OA.8 Solve two-step word problems using the four operations. Represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. (This standard is limited to problems posed with whole numbers and having whole-number answers; students should know how to perform operations in the conventional order when there are no parentheses to specify a particular order, i.e., Order of Operations.)
- 3.OA.9 Identify arithmetic patterns (including patterns in the addition table or multiplication table), and explain them using properties of operations. *For example, observe that 4 times a number is always even, and explain why 4 times a number can be decomposed into two equal addends.*
- 3.NBT.3 Multiply one-digit whole numbers by multiples of 10 in the range 10–90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

Instructional Days Recommended: 3

In Lesson 19, students initially use the place value chart to multiply by multiples of 10. To solve 2×40 , for example, they begin by modeling 2×4 in the ones place. Students relate this to multiplying 2×4 tens, locating the same basic fact in the tens column. They see that when multiplied by 10, the digits in the product shift one place value to the left. Complexities are addressed as regrouping becomes involved with problems like 4×6 , where the product has mixed units of tens and ones. However, the same principle applies – the digits shift once to the left.

Lesson 20 carries students' understanding from Lesson 19 to more abstract situations using a wider range of multiples of 10. Students learn to model place value strategies using the associative property, e.g., $2 \times 30 = 2 \times (3 \times 10)$, and $4 \times 60 = 4 \times (6 \times 10) = (4 \times 6) \times 10$. In Lesson 21, students apply learning from Topic F to solving two-step word problems and multiplying single-digit factors and multiples of 10. They use the rounding skills learned in Module 2 to estimate and assess the reasonableness of their solutions.

**The sample homework responses contained in this manual are intended to provide insight into the skills expected of students and instructional strategies used in Eureka Math.*

Lesson 19

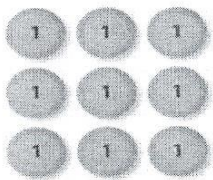
Objective: Multiply by multiples of 10 using the place value chart.

Homework Key

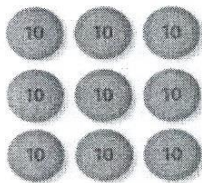
- 9; 9
 - 9; 90
- 10; 10
 - 10; 100
 - 25; 25
 - 25; 250
- Products matched to corresponding solutions
- 240; tape diagram models equation.

Homework Samples

1. Use the disks to complete the blanks in the equations.

a. 

$3 \times 3 \text{ ones} = \underline{9} \text{ ones}$
 $3 \times 3 = \underline{9}$

b. 

$3 \times 3 \text{ tens} = \underline{9} \text{ tens}$
 $30 \times 3 = \underline{90}$

2. Use the chart to complete the blanks in the equations.

| tens | ones |
|------|----------------|
| | ●●●●● ●●●●● |

a. $2 \times 5 \text{ ones} = \underline{10} \text{ ones}$
 $2 \times 5 = \underline{10}$

| tens | ones |
|----------------|------|
| ●●●●● ●●●●● | |

b. $2 \times 5 \text{ tens} = \underline{10} \text{ tens}$
 $2 \times 50 = \underline{100}$

| tens | ones |
|------|----------------------------------|
| | ●●●●● ●●●●● ●●●●● ●●●●● |

c. $5 \times 5 \text{ ones} = \underline{25} \text{ ones}$
 $5 \times 5 = \underline{25}$

| tens | ones |
|----------------------------------|------|
| ●●●●● ●●●●● ●●●●● ●●●●● | |

d. $5 \times 5 \text{ tens} = \underline{25} \text{ tens}$
 $5 \times 50 = \underline{250}$

Lesson 20

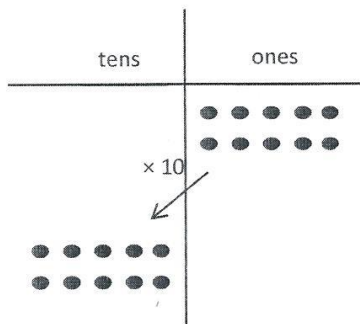
Objective: Use place value strategies and the associative property $n \times (m \times 10) = (n \times m) \times 10$ (where n and m are less than 10) to multiply by multiples of 10.

Homework Key

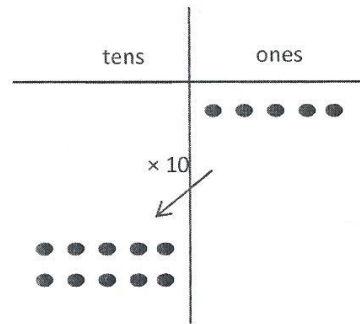
- 100
 - 100
 - 20; 200
 - 5; 200
- 60
 - 9; 90
 - 12; 120
 - 15; 150
- Explanations will vary.

Homework Samples

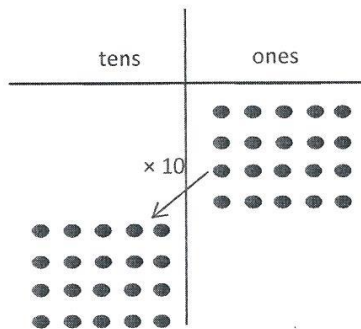
- Use the chart to complete the equations. Then, solve.



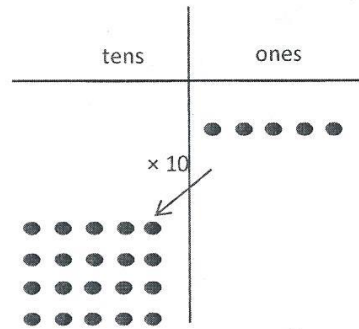
a. $(2 \times 5) \times 10$
 $= (10 \text{ ones}) \times 10$
 $= \underline{100}$



b. $2 \times (5 \times 10)$
 $= 2 \times (5 \text{ tens})$
 $= \underline{100}$



c. $(4 \times 5) \times 10$
 $= (\underline{20} \text{ ones}) \times 10$
 $= \underline{200}$



d. $4 \times (5 \times 10)$
 $= 4 \times (\underline{5} \text{ tens})$
 $= \underline{200}$

Lesson 21

Objective: Solve two-step word problems involving multiplying single-digit factors and multiples of 10.

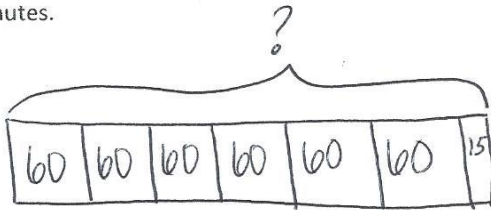
Homework Key

1. 375 minutes; solution includes model and equation with unknown
2. 210; solution includes model and equation with unknown
3. Yes; explanations will vary; solution includes model and equation with unknown
4. 23; solution includes model and equation with unknown
5. No; explanations will vary; solution includes model and equation with unknown
6. \$450; solution includes model and equation with unknown

Homework Samples

Use the RDW process for each problem. Use a letter to represent the unknown.

1. There are 60 minutes in 1 hour. Use a tape diagram to find the total number of minutes in 6 hours and 15 minutes.



$$(6 \times 60) + 15 = m$$

$$360 + 15 = m$$

$$375 = m$$

There are 375 minutes in 6 hours and 15 minutes.